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APPLICATION OF THE QUASI-NEWTON IN INTERIOR POINT METHODS FOR SOLVING THE PREDISPATCH PROBLEM

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Abstract

Brazilian energy matrix is essentially based on hydroelectricity with long transmission lines, allowing the exchange of energy produced in all regions of the country, the increased demand for energy and the search for lower costs, the application of more efficient and robust methods to minimize generation and transmission losses is necessary, since these are functions of generated and transmitted power, respectively. The purpose of this work is to implement primal dual interior point method for the predispatch of a hydroelectric system, with partial replacement of the Newton method with the quasi-Newton method in order to compute the system Jacobian matrix and reduce the computational costs of the iterations arising from approximations of the inverses of the Hessian matrix. This means that, in order to obtain a search direction, only a matrix vector product is necessary, which is much more efficient, for example, than the Newton method, in which a linear system has to be solved at each iteration. Computational results prove the efficiency of the approach used.
Keywords: linear programming, interior point methods, initial point.

1 Introduction

Brazilian electric system is unique in the world. Its flexibility even enables the demand for energy to grow before the offer. This flexibility results from the system being unique and its large shared hydraulic reserves, with interconnected reservoirs for the storage of energy. Furthermore, it has one of the lowest operation and environmental costs in the world [22].

In Brazil, energy production is predominantly hydraulic, with thermal generation having a supplementary function during peaks in demand. Use of the great majority

of generating plants water reservoirs is planned in order to get the most out of the pluviometric diversity in the different existing catchment areas. Thus, the possibility of interconnecting catchment areas located in different geographic regions provides major energy gains for the Brazilian system, because, accordingly, it is possible to take advantage of the different seasons and pluviometric levels.

In the predispatch of the hydroelectric systems, the generating units have a target to meet on certain days, established by long-term planning, and, as this planning serves as an operational directive, it requires a more detailed representation of the operation of the system, in which hydroelectric and thermoelectric plants are represented at the turbine/generator level and all the relevant restrictions of the generation and transmission system must be taken into account. An aggravating factor is that most of the hydroelectric generation units are located far from the main consumer centers; consequently, an extensive transmission network is necessary in order to interconnect the most distant energy generation and consumption points, resulting in significant losses in these networks.

Predispatch provides the energy targets for each day and thus, in order to fulfill them, it is necessary to represent the electric power system for each of the 24 hours of the day, detailing the operational conditions of the system. Optimizing this system is a fairly complex task, and thus to try to fulfill the necessities and criteria that the hydro resources possess, it is necessary to apply methods that minimize costs at all stages of generation, as well as transmission losses.

The methodology used to develop this work is the primal-dual interior point method, because it presents satisfactory results for optimal power flow problems [20], [4]. This methodology comprises the application of the Newton method to the optimality condition of the problem, thus aiming at solution of non-linear systems; however, solving the linear systems that arise from this approach is, computationally, a more expensive task than an interior point method [11].

The contribution of this work is the implementation of interior point methods for the predispatch of the hydroelectric system, with the replacement of the Newton method with the Broyden's quasi-Newton method in order to compute the Jacobian matrix of the system, aimed at the low computational costs of the iterations arising from the approximations of the inverses of the Hessian matrix. This means that, in order to obtain the search direction, only a matrix vector product is necessary, which is much more efficient, for example, than the Newton method, in which a linear system has to be solved at each iteration. Application of this methodology to predispatch problems is innovative.

2 Quadratic programming

The interest in the study of the minimization of quadratic functions resides in the large number of applications that fall into this format. This type of problem is considered one of the simplest in the non-linear optimization area [7]. It often appears as a sub-problem to assist in the solution of more complex non-linear problems.

A quadratic programming problem may be formulated as follows [8]:

$$\begin{array}{ll} \min & \frac{1}{2}x^T Gx + c^T x \\ \text{s.t} & Ax \leq b \\ & x \geq 0. \end{array} \tag{2.1}$$

where G is a symmetric square matrix $(n \times n)$, c is an n-dimensional vector, whose components are the coefficients of the linear terms in the objective function. If $G = 0$, the objective function is linear, thus linear programming can be seen as a special case of quadratic programming.

In this work, the objective function is quadratic with separable variables and the procedure for developing the primal-dual-type interior-point methods for quadratic programming is essentially the same as that used in linear programming.

2.1 Interior Point Methods

The interior-point methods attempt to find an optimal solution for a linear programming problems by moving through the interior of the positive orthant [24, 25]. This comprises the application of the Newton method to the optimality conditions of the problem [25, 15, 16].

Consider:

$$F(x, y, z) = \begin{bmatrix} b - Ax \\ c - A^T y - z \\ XZe \end{bmatrix}, \tag{2.2}$$

where X and Z are diagonal matrices, formed by the x and z vectors, respectively, and e is a vector with all the elements equal to one. Consider the residuals:

$$F(x, y, z) = - \begin{bmatrix} r_p \\ r_d \\ r_a \end{bmatrix}. \tag{2.3}$$

From an initial point (x^0, y^0, z^0), the point (x^1, y^1, z^1), is computed in the following manner:

$$(x^1, y^1, z^1) = (x^0, y^0, z^0) - \left[J(x^0, y^0, z^0)\right]^{-1} \; F(x^0, y^0, z^0), \tag{2.4}$$

where the Jacobian is given by:

$$J(x, y, z) = \begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ Z & 0 & X \end{bmatrix}. \tag{2.5}$$

In general $(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k, y^k, z^k) + d^k$, is defined, where in the d^k step is given by:

$$d^k = \begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ Z^k & 0 & X^k \end{bmatrix}^{-1} \begin{bmatrix} r_p^k \\ r_d^k \\ r_a^k \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}. \tag{2.6}$$

In order to ensure that the point remain interior, this step is multiplied by a constant α_k, called "step size". Thus, after applying the Newton method to the optimality conditions, we get:

$$(x^{k+1}, y^{k+1}, z^{k+1}) = (x^k, y^k, z^k) + \alpha^k (dx^k, dy^k, dz^k). \tag{2.7}$$

The primal and dual steps, respectively, are computed as follows:

$$\alpha_p = \min\left\{-(x_i)^k/(dx_i)^k; dx_i^k < 0\right\} \quad \alpha_d = \min\left\{-(z_i)^k/(dz_i)^k; dz_i^k < 0\right\}, \tag{2.8}$$

where in ${\alpha_1}^k = \min\{\alpha_p^k, \alpha_d^k\}$, results in $\alpha^k = \min\left\{\tau\alpha_1^k, 1\right\}$ where $\tau \in (0, 1)$ for linear programming. In practice, using different steps for primal and dual variables, we get better computational results.

3 The Predispatch Problem

Predispatch is a short-term operational problem that is intended to define the best alternatives for hydroelectric generation, transmission of energy and thermal generation for one-week, or even single-day, horizons. The aim is to fulfill demand and achieve energy production targets, defined in long-term planning problem.

Flow restrictions may be divided into blocks that are repeated over each time interval, representing the electric system in these intervals. Thus, we get an independent formulation of Kirchhoff's laws for each interval. In the network flow model, power

flows enable consideration of transmission limits as restrictions; transmission losses are characterized as a performance criterion [18].

3.1 Static Model

The power system problem optimization of generation and transmission costs with m bars, n lines and g generators can be modeled using the following formula [10]:

$$\begin{aligned} \min\ & \alpha\frac{1}{2}[f^T R f] + \beta\frac{1}{2}[p^T Q p + c^T p], \\ \text{s.t } & Af = Ep - d,\ Xf = 0, \\ & f^{min} \leq f \leq f^{max},\ p^{min} \leq p \leq p^{max}, \end{aligned}$$

where:

- $f \in \mathbb{R}^{n\times 1}$ represents the active power flow;
- $p \in \mathbb{R}^{g\times 1}$ represents the active power generation;
- $Q \in \mathbb{R}^{g\times g}$ represents the quadratic component of the generation cost;
- $R \in \mathbb{R}^{n\times n}$ represents the diagonal matrix of line resistance;
- $d \in \mathbb{R}^{m\times 1}$ represents the active power demand;
- $X \in \mathbb{R}^{n-m+1\times n}$ represents the line reactance matrix;
- E represents a matrix of order $m \times g$ with each column containing exactly one element equal to 1 and the other elements null;
- $A \in \mathbb{R}^{m\times n}$ represents the incidence matrix of the transmission network;
- $c \in \mathbb{R}^{g\times 1}$ represents the linear component of the generation cost;
- f^{max}, $f^{min} \in \mathbb{R}^{n\times 1}$, p^{max} and $p^{min} \in \mathbb{R}^{g\times 1}$ are the flow and generating limits for active power, respectively;
- α and β are weights of the objectives to be minimized;
- $q \in \mathbb{R}^{g\times 1}$ represents the energy generation target established by long-term planning.

For this model, both components of the objective function are quadratic with separable variables. The first component represents the value of transmission losses. The second component characterizes the generation cost of the plants [23].

3.2 Dynamic Model

The static model results in representation of the problem for a single point in time. The dynamic model is intended to find the best generation alternatives for t-interval time horizons. Thus, we get t-subproblems with the same structure as the static model, coupled with additional constraints.

In other words, the predispatch problem is a dynamic flow model of optimal power, which can be represented in the following manner[20, 21]:

$$\min \frac{\alpha}{2} \sum_{k=1}^{t} [(f^k)^T R^k f^k] + \frac{\beta}{2} \sum_{k=1}^{t} [(p^k)^T Q^k p^k + c^T p^k], \tag{3.1}$$

where

$$\begin{aligned} A^k f^k - E p^k &= -d^k, \quad \forall \quad k = 1, ..., t, \\ X^k f^k &= 0, \quad \forall \quad k = 1, ..., t, \\ f^{min} \leq f^k &\leq f^{max}, \quad \forall \quad k = 1, ..., t, \\ p^{min} \leq p^k &\leq p^{max}, \quad \forall \quad k = 1, ..., t, \\ \sum_{k=1}^{t} p^k &= q, \end{aligned} \tag{3.2}$$

where $q \in \mathbb{R}^{g \times 1}$ represents the energy generation target established by long-term planning.

Optimality conditions for problem [3.2, Subsection 3.2] may be decomposed into primal feasibility, dual feasibility and complementarity constraints, detailed below.

- Primal feasibility

$$B\widetilde{f}^k - \widehat{E}\widetilde{p}^k = \widehat{d}^k, \quad \forall \quad k = 1, ..., t, \tag{3.3}$$

$$\widetilde{f}^k + s_{\widetilde{f}}^k = \widetilde{f}^{max}, \quad \forall \quad k = 1, ..., t, \tag{3.4}$$

$$\widetilde{p}^k + s_{\widetilde{p}} = \widetilde{p}^{max}, \quad \forall \quad k = 1, ..., t \tag{3.5}$$

$$\sum_{k=1}^{t} \widetilde{p}^k = \widetilde{q}, \tag{3.6}$$

$$(\widetilde{f}^k, s_{\widetilde{f}}^k, \widetilde{p}, s_{\widetilde{p}}^k) \geq 0. \tag{3.7}$$

- Dual feasibility

$$B^T y_{\widetilde{f}}^k - w_{\widetilde{f}}^k - R\widetilde{f}^k + z_{\widetilde{f}}^k = c_{\widetilde{f}}, \quad \forall \ \ k = 1, ..., t, \tag{3.8}$$

$$-\widehat{E}^T y_{\widetilde{f}}^k - w_{\widetilde{p}}^k + y_a - Q\widetilde{p}^k + z_{\widetilde{p}}^k = c_{\widetilde{p}}, \quad \forall \ \ k = 1, ..., t, \tag{3.9}$$

$$(z_{\widetilde{f}}^k, w_{\widetilde{f}}^k, z_{\widetilde{p}}^k, w_{\widetilde{p}}^k) \geq 0, \ y_{\widetilde{f}}^k, \qquad y_a \quad free. \tag{3.10}$$

- Complementarity conditions

$$\widetilde{F}^k Z_{\widetilde{f}}^k e = 0, \tag{3.11}$$

$$S_{\widetilde{f}}^k W_{\widetilde{f}}^k e = 0, \tag{3.12}$$

$$\widetilde{P}^k Z_{\widetilde{p}}^k e = 0, \tag{3.13}$$

$$S_{\widetilde{p}}^k W_{\widetilde{p}}^k e = 0. \tag{3.14}$$

$$\tag{3.15}$$

3.3 Newton Method

Applying the Newton method to optimality conditions, we get:

$$Jd = r, \tag{3.16}$$

where

$$J = \begin{bmatrix} M_1 & 0 & 0 \dots & 0 \\ 0 & M_2 & 0 \dots & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & M_t \\ I_{\widetilde{p}}^1 & I_{\widetilde{p}}^2 & \dots & I_{\widetilde{p}}^t \end{bmatrix},$$

each M_k corresponds to the following matrix:

$$\begin{bmatrix} B & -\widehat{E} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ -R & 0 & 0 & 0 & B^T & -I & I & 0 & 0 & 0 \\ 0 & -Q & 0 & 0 & -\widehat{E}^T & 0 & 0 & -I & I & I \\ Z^k_{\widetilde{f}} & 0 & 0 & 0 & 0 & 0 & \widetilde{F}^k & 0 & 0 & 0 \\ 0 & 0 & W^k_{\widetilde{f}} & 0 & 0 & S^k_{\widetilde{f}} & 0 & 0 & 0 & 0 \\ 0 & Z^k_{\widetilde{p}} & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{P}^k & 0 \\ 0 & 0 & 0 & W^k_{\widetilde{p}} & 0 & 0 & 0 & S^k_{\widetilde{p}} & 0 & 0 \end{bmatrix},$$

whose directions are

$$d^T = \begin{bmatrix} (d\widetilde{f}^k)^T & (d\widetilde{p}^k)^T & (ds^k_{\widetilde{f}})^T & (ds^k_{\widetilde{p}})^T & (dy^k_{\widetilde{f}})^T & (dw^k_{\widetilde{f}})^T & (dz^k_{\widetilde{f}})^T & (dw^k_{\widetilde{p}})^T & (dz^k_{\widetilde{p}})^T & (dy_a)^T \end{bmatrix},$$

and r represents the residual vector:

$$r_1 = \widehat{d}^k - B\widetilde{f}^k + \widehat{E}\widetilde{p}^k, \tag{3.17}$$
$$r_2 = \widetilde{f}^{max} - \widetilde{f}^k - s^k_{\widetilde{f}}, \tag{3.18}$$
$$r_p = \widetilde{p}^{max} - \widetilde{p}^k - s^k_{\widetilde{p}}, \tag{3.19}$$
$$r_y = c_{\widetilde{f}} - B^T y^k_{\widetilde{f}} + w^k_{\widetilde{f}} + R\widetilde{f}^k - z^k_{\widetilde{f}}, \tag{3.20}$$
$$r_g = c_{\widetilde{p}} + \widehat{E}^T y^k_{\widetilde{f}} + w^k_{\widetilde{p}} - y_a + Q\widetilde{p}^k - z^k_{\widetilde{p}}, \tag{3.21}$$
$$r_{zf} = \mu e - \widetilde{F}^k Z^k_{\widetilde{f}} e, \tag{3.22}$$
$$r_{wf} = \mu e - S^k_{\widetilde{f}} W^k_{\widetilde{f}} e, \tag{3.23}$$
$$r_{zp} = \mu e - \widetilde{P}^k Z^k_{\widetilde{p}} e, \tag{3.24}$$
$$r_{wp} = \mu e - S^k_{\widetilde{p}} W^k_{\widetilde{p}} e, \tag{3.25}$$
$$r_m = \widetilde{q} - \sum_{k=1}^{t} \widetilde{p}^k. \tag{3.26}$$

Thus, we get the following equation system formed by the Newton directions [3]:

$$\begin{cases} B d\widetilde{f}^k - \widehat{E} d\widetilde{p}^k = r_1, \\ d\widetilde{f}^k + ds^k_{\widetilde{f}} = r_f, \\ d\widetilde{p}^k + ds^k_{\widetilde{p}} = r_p, \\ B^T dy^k_{\widetilde{f}} - dw^k_{\widetilde{f}} - R d\widetilde{f}^k + dz^k_{\widetilde{f}} = r_y \\ -\widehat{E}^T dy^k_{\widetilde{f}} - dw^k_{\widetilde{p}} + dy_a - Q d\widetilde{p}^k + dz^k_{\widetilde{p}} = r_g, \\ \widetilde{F}^k dz^k_{\widetilde{f}} + Z^k_{\widetilde{f}} d\widetilde{f}^k = r_{zf}, \\ S^k_{\widetilde{f}} dw^k_{\widetilde{f}} + W_{\widetilde{f}} ds^k_{\widetilde{f}} = r_{wf}, \\ \widetilde{P}^k dz^k_{\widetilde{p}} + Z^k_p d\widetilde{p}^k = r_{zp}, \\ S^k_{\widetilde{p}} dw^k_{\widetilde{p}} + W^k_{\widetilde{p}} ds^k_{\widetilde{p}} = r_{wp}, \\ \sum_{k=1}^{t} d\widetilde{p}^k = r_m. \end{cases} \tag{3.27}$$

This system may be considerably reduced by replacing the variables and obtaining the system [3.28, Subsection 3.3] to be solved [3].

$$\sum_{k=1}^{t} [(D^k_{\widetilde{p}})^{-1} - (D^k_{\widetilde{p}})^{-1} \widehat{E}^{T^{-1}} \widehat{E} (D^k_{\widetilde{p}})^{-1}] dy_a = r_m + \sum_{k=1}^{t} (D^k_{\widetilde{p}})^{-1} [r_b + \widehat{E}^T M^{-1} r], \tag{3.28}$$

whose direct solution requires great computation effort, because we lose sparsity and $M = B(D^k_{\widetilde{f}})^{-1} B^T + D^k$ has a dimension equal to the number of lines plus one $(n+1)$ and dy_a has the number of generators as the dimension. For example, for the Brazilian system with 3,511 bars, studied in this work, the matrix M has the dimension $(4,238 \times 4,238)$ and dy_a is the vector with dimension (265×1). Therefore, the direct approach for solving this system in large scale problems should be discarded [5].

It is important to emphasize that the solution for the system [3.28, Section 3.3] is the stage that requires the greatest computation effort to solve the optimal power flow problem, modeled as described in this work. The objective comprises an exploration of alternatives for solving this problem in the best manner possible. This consideration is even more critical in the approach to the pre-dispatch problem, because there are many matrices that must be decomposed in the system to be solved, as many as the number of intervals defined on the study horizon (in this article, there will be 24 intervals, representing the hours of the day).

4 Quasi-Newton Methods

The interior point methods are robust and widely-used methods for solving large-scale problems, because they present fast convergence. Accordingly, it is intuitive to think of the replacement of the Newton Method with the Quasi-Newton Method, where an approximation to the Jacobian matrix is used instead of making an exact computation as in the Newton method.

The Newton method is excellent for solving non-linear systems; however, it involves a high computational cost, and so a relatively satisfactory method, but with a lower computational cost, is an interesting proposition. Most of the quasi-Newton methods were developed with this purpose [17, 14]. The purpose is to generate a x_k sequence, where:

$$B_k d_k = -F(x_k). \tag{4.1}$$

Where:

$$x_{k+1} = x_k + d_k. \tag{4.2}$$

A very highly-regarded quasi-Newton method is the secant method [12], where the approximation matrices for the Jacobian matrix are chosen in a manner that satisfies the secant equation [4, Section 4]. The BFGS method developed by Broyden - Feltcher - Goldfarb - Shanno used in this work is a quasi-Newton method, often used to solve non-linear systems, whose formula for B_{k+1} comprises a correction of rank one on the matrix B_k; thus B_{k+1} is the orthogonal projection of B_k in the set of matrices that satisfy the secant equation. Given B_k, a new approximation of B_{k+1} to J_{k+1} is given by [11]:

$$B_{k+1} d_k = F(x_{k+1}) - F(x_k), \tag{4.3}$$

$$B_{k+1} = B_k + \frac{[(F(x_{k+1}) - F(x_k)) B_k s_k)] s_k^T}{s_k^T s_k}, \tag{4.4}$$

where $s_k = x_{k+1} - x_k$.

For the computation of the Hessian matrix $H_{k+1} = B_{k+1}^{-1}$ we will use the Sherman-Morrison-Woodbury formula [9].

$$H_{k+1} = H_k + \frac{(s_k - H_k(F(x_{k+1}) - F(x_k))s_k^T H_k}{s_k^T H(F(x_{k+1}) - F(x_k))}. \tag{4.5}$$

The Hessian matrix for predispatch problems is will conditioned, and may result in convergence problems. In order to remedy this difficulty every time this matrix is very ill conditioned, the Newton step will be performed in this iteration; i.e., the usual interior-point method. In practice this procedure works as a corrective measure, and for large-scale problems, it is necessary to apply it once or twice at the most.

5 Computational Results

Implementation was developed in MATLAB R2018, with a precision of 10^{-6}, in order to consider the optimality conditions of the problem satisfied. The following equipment was used: IMAC retina 5k 2019, Processing Speed 3,4 GHz intel core i5 Quad-Core with RAM 8 GB DDR4, MacOs Ventura 13.1(22C65).

The networks on which the tests were performed include the IEEE30 system, shown in Figure [1, Section 5], and the IEEE118 system, representing the United States Midwest. The Brazilian South-Southeast-Midwest system, with 1,654 (SSECO1654) bars, and the Real Brazilian Interconntected System, comprising 1,993 bars.

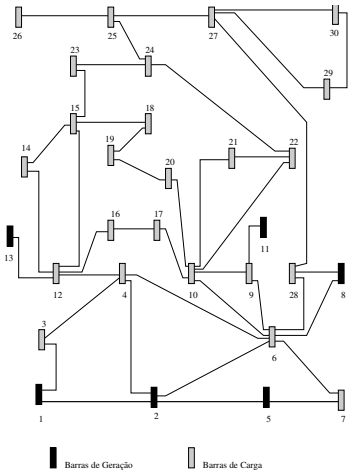

Figura 1: IEEE System

The tests performed use the initial point shown in the Equation [5.6, Section 5]. This presented satisfactory results in previous experiments [4, 2, 19, 20]. The initial Jacobian matrix is computed using the Newton method.

$$f^0 = \frac{f^{max}}{2}, \tag{5.1}$$

$$p^0 = \frac{p^{max}}{2}, \tag{5.2}$$

$$y_1^0 = y_2^0 = y_3^0 = y_4^0 = 0, \tag{5.3}$$

$$z_1^0 = w_1^0 = (R + I)e, \tag{5.4}$$

$$z_2^0 = w_2^0 = e, \tag{5.5}$$

$$z_3^0 = w_3^0 = e. \tag{5.6}$$

Tabela 1: Comparison Classic MPI and Quasi-Newton MPI

	Classic MPI		Quasi-Newton MPI	
Systems	Time(s)	Iteration	Time(s)	Iteration
IEEE30	0.1896	3	fails to converge	-
IEEE118	1.9524	6	fails to converge	-
SSECO1654	40.1014	14	67.2379	16
BRASIL1993	44.2235	9	63.1505	9

In Table 1, there is a comparison between the classic primal-dual interior point methods; i.e., with the use of only the Newton Method for solving the non-linear system with the primal-dual interior point method with a simple direct replacement by the Broyden's quasi-Newton method for solving the system. In this table, it can be perceived that the classic interior-point method is more efficient for all the systems, when compared to the method that uses the quasi-Newton; this is due to the fact that the when the inverse Jacobian matrix is computed using the Sherman-Morrison-Woodbury formula, it was ascertained that it is extremely poorly conditioned, and it is close to being singular at several timepoints, thus resulting in a convergence problem.

In order to remedy this difficulty, a preconditioner is applied in the $Jd = r$ system; i.e., we will apply techniques that consist of modifying the original system to an equivalent one, whose matrix is better conditioned, and the new system will have better spectral properties, such as eigenvalues close to the unit.

Methodology used has been applied with great success in [6] was suggested by Manteuffel [13], with a hybrid approach [1]. In this method, a global increment in the

Tabela 2: Comparison Classic MPI and Preconditioned Quasi-Newton MPI

	Classic MPI		Preconditioned Quasi-Newton MPI	
Systems	Time(s)	Iteration	Time(s)	Iteration
IEEE30	0.1896	3	0.1619	3
IEEE118	1.9524	6	1.4366	5
SSECO1654	40.1014	14	36.3244	16
BRASIL1993	44.2235	9	33.9870	9

coefficient matrix diagonal is made. Thus, incomplete factorization is applied in the $J = J + \alpha.diag(J)$ matrix in which α Ãs positive number and $diag(J)$ denotes the diagonal part of the Jacobian matrix. The computation of $\alpha_i = 6 \cdot 10^{-4} \cdot 2^i$ with $i = 1, 2, ..., 15$, in which i is the number of factorization restarts.

Every time the Jacobian matrix has a very high condition number, an interior point iteration would be performed with the Newton method, and in this case, it would be used as a corrective measure.

Observe in Table 2 that, with the preconditioner, all the systems converge more quickly than when compared to classic interior-point methods. For the IEEE118 and SSECO1654 systems with the pre-conditioning application, the number of iterations is higher, and the explanation is that the linear system solution is approximated, but the computational cost is still lower, because it is no longer necessary to solve a linear system, but rather by simply performing a matrix vector product.

The incomplete LU preconditioner with the hybrid approach obtained satisfactory results for the IEEE30 problems and the actual Brazilian system; however, the approach was not sufficient to obtain convergence with the IEEE118 and the Brazilian South-Southeast and Midwest systems. In these cases, it was also necessary to perform a classic interior-point method iteration; i.e., when obtaining the Jacobian matrix using the Newton method, whenever the matrix became poorly conditioned. For the IEEE118 system, this corrective measure was necessary a single time and, for the Brazilian system, it was necessary to perform the measure twice. Analyzing the convergence, it was noted that these corrective measures were always necessary close to an optimal solution.

In Figure [2, Section 5], it is possible to ascertain that, for the Brazilian interconnected system, all the generators worked at their limits at peak times to fulfill the energy demand in the system. Similar graphs were obtained for other systems, thus showing that the energy dispatch during the 24-hour period analyzed was coherent.

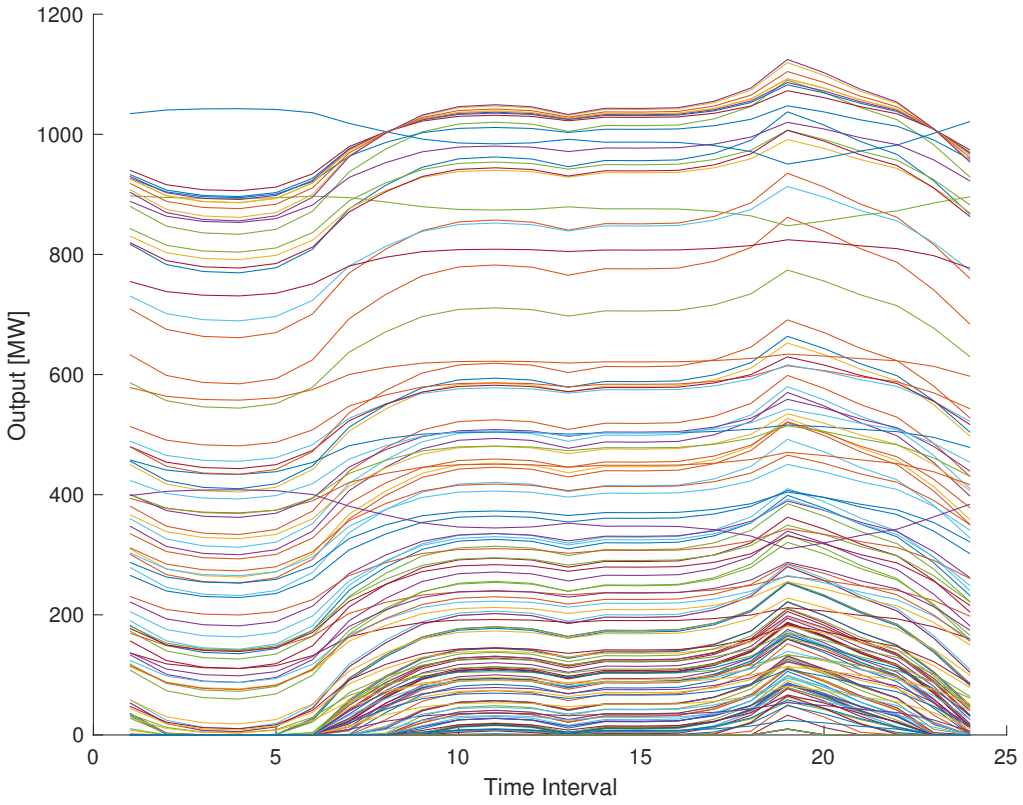

Figura 2: Dispatch of the generators of the Brazilian interconnected system - BRASIL1993

6 Conclusions

Brazilian energy matrix is essentially based on hydroelectricity with long transmission lines, allowing the exchange of energy procuded in all regions of the country, the increased demand for energy and the search for lower costs, the application of more efficient and robust methods to minimize generation and transmission losses is necessary, since these are functions of generated and transmitted power, respectively.

In this paper, the primal-dual interior point methods were used to solve a predispatch problem in a hydrothermal system. The contribution of this research is to solve this problem by replacing the Jacobian matrix, obtained by the Newton method, by an approximation, in order to obtain a reduction in computational costs.

One of the main concerns when implementing interior-point methods is in the solution of the Newton equation systems, present at each iteration of this method, and this is the step that consumes most of the processing time, and so it should be executed efficiently. The fundamental idea of the quasi-Newton methods is to use an iterative process for an approximation to the Jacobian matrix, instead of performing an exact computation, such as in the Newton method. In this work, Broyden's quasi-Newton method was used for approximation to the Jacobian matrix at interior point methods and the Sherman-Morrison-Woodbury was used to compute the Hessian matrix, and it was ascertained that it is extremely poorly conditioned, resulting in convergence problems for the method. To solve this problem, a preconditioner was also implemented, in order to obtain a more stable Hessian matrix.

The incomplete LU preconditioner with a hybrid approach obtained satisfactory results for the IEEE30 problems and the actual Brazilian system; however, the approach was sufficient to obtain convergence for the IEEE118 systems and the Brazilian system in the South-Southeast and Midwest systems. For these cases, it was also necessary to perform an iteration of classic interior point methods; i.e., when obtaining the Jacobian matrix using the Newton method, whenever the matrix became poorly conditioned. For the IEEE118 system, this corrective measure was necessary a single time and, for the Brazilian system, it was necessary twice.

The computational tests showed that the approach used is quite efficient in order to solve the optimal power flow problems. The results obtained demonstrated the adequacy of the methodology, both in terms of numerical aspects and in computational time.

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