REVISEM

Revista Sergipana de Matemática e Educação Matemática

https://doi.org/10.34179/revisem.v9i4.20708

INFLECTION POINTS IN PROCESS OF MATH INSTITUTIONALIZATION IN SOCIETY

SALTOS NO PROCESSO DE INSTITUCIONALIZAÇÃO DA MATEMÁTICA NA SOCIEDADE

João Paulo Attie
Universidade Federal de Sergipe – UFS
jpattie@mat.ufs.br

Manoel Oriosvaldo de Moura
Universidade de São Paulo – USP
modmoura@usp.br

Abstract

In this paper, we point out what we consider to be the four fundamental historical milestones responsible for the current importance of mathematics for society. To identify these points, we carried out bibliographical research of a theoretical nature, which was made possible by a literature review, in which we followed some specific steps, such as selecting the material, reading and analyzing the texts, recording the material found and, finally, a final analysis of the information to prepare the article. The milestones found were the construction of Deductive Geometry, the development of Symbolic Algebra and Analytical Geometry, the structuring of Calculus and the alliance between Symbolic Logic and Technology. In the end, we highlight the paradox that is being consolidated in this historical process, that, even though the presence of mathematical knowledge in institutions, in everyday life and in the production of goods is increasing, the invisibility of this type of knowledge in these areas is also increasing. same spaces, that is, in institutions, in everyday life and in the production chain.
Keywords: Importance of Mathematics. Mathematical Knowledge. Social History of Mathematics. Math Institutionalization.

Resumo

Neste artigo, apontamos o que consideramos ser quatro divisas históricas fundamentais responsáveis pela importância atual da matemática para a sociedade. Para identificar esses pontos, realizamos uma pesquisa bibliográfica de cunho teórico, que foi possível por meio de uma revisão bibliográfica, na qual seguimos algumas etapas particulares, como a seleção dos textos, a leitura e análise desse material, o registro do que foi selecionado e, ao final, uma análise dos dados e informações para a elaboração deste artigo. As divisas encontradas foram a edificação da Geometria Dedutiva, o desenvolvimento, em paralelo, da Álgebra Simbólica e da Geometria Analítica, a construção do Cálculo e a associação entre a Lógica Simbólica e a Tecnologia. Ao final, destacamos o paradoxo que está se consolidando dentro desse processo histórico, de que, ainda que a presença do conhecimento matemático seja crescente em instituições, no cotidiano e na produção de serviços e de bens, também está aumentando a invisibilidade desse tipo de conhecimento nesses mesmos espaços.
Palavras-chave: Importância da Matemática; Conhecimento Matemático; História Social da Matemática.

INTRODUCTION

Where does the importance given to Mathematics in our culture come from? Despite the existence of an almost universal discourse that Mathematics is in everything, we cannot consider that a satisfactory answer since it does not provide evidence to support it. From this discussion, we seek, in this paper, to historically analyze the development of this importance from eminently bibliographical and analytical research. After all, why is mathematics considered so important? A curious phenomenon that we point out is that, allied to this importance, which we believe to be unquestionable, reveals enormous invisibility concerning mathematical knowledge.

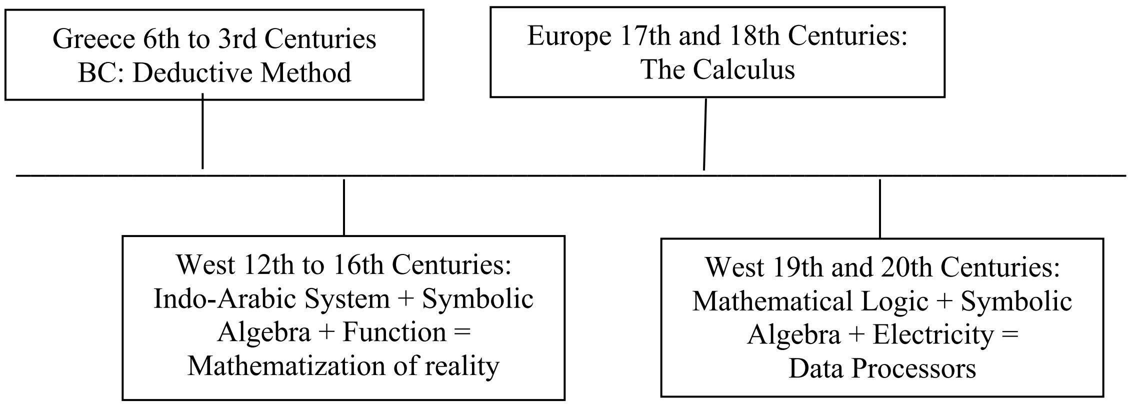
Historically, the process of valuing mathematical knowledge in the social environment can be described through some qualitative leaps in the history of its development, in which fundamental advances in this field have enabled considerable changes in relation to the economy, technology and society. Despite the influence that mathematical knowledge has on systems of thought, such as rationalism, for example, we have opted in this paper to establish these milestones solely from elements intrinsic to Mathematics itself.

These leaps, or inflection points, however, did not occur suddenly, in a visible or determined way, like a revolution or a seizure of power, but, on the contrary, they developed and consolidated patiently, slowly, over periods of varying lengths.

The structuring process of the deductive demonstration, in Greece, in the middle of the 6th century BC, the development of Symbolic Algebra and Analytical Geometry, especially in Italy and France during the Renaissance, the construction of Differential and Integral Calculus and Analysis, especially in France, England and Germany, from the 17th century and the union between Symbolic Logic and Technology, in the 19th and 20th centuries, culminating with the emergence and invasion of data processors, can be pointed out as the fundamental historical moments that permeate the process of expanding the importance that mathematical knowledge has been establishing throughout human history. We consider it important to point out that the authors, aware of their particularity in the space and culture to which they belong, start from a Eurocentric perspective. As should be evident, the use of sources other than those centered on the West could provide another type of perspective.

We consider it important to present, before the discussion, a timeline, pointing out the periods highlighted in the research.

Figure 1: Timeline



Source: Authors, 2024.

METHODOLOGY

We conducted theoretical bibliographic research, which was made possible by a literature review, having as main sources historians of mathematics, such as Bell (2003), Eves (2005), and Boyer (1974), mathematicians such as Caraça (1958), Whitehead (2006) and Dieudonné (1990) and also historians of science, such as Crosby (1997) and Dantzig (1970). We assume that we did not use primary sources, which may open a discussion about the role of the choices of sources used and their limitations based on their cultural and temporal background. We follow some specific steps according to a work of this nature, such as the selection of the material, the reading and analysis of the texts, the documenting of the material found and, finally, a final analysis of the information for the elaboration of the essay. The selection of materials followed criteria such as the work having academic relevance and dealing, in some way, with the relationship between the development of mathematics and its influences on society, science or technology. The reading was initially carried out in a floating manner, so that the categories could take shape, and, in a second stage, in a more incisive manner, in search of the desired relationship. Regarding the analysis, in the course of the work, one of the central elements we sought was identifying the most prominent points in the development of mathematical knowledge. This identification can be attributed according to the existence of strong elements, found in bibliographic research, such as the influence on the development of science, its applications in technology and the influence itself on society.

RESULTS

The deductive method of the Greeks marks the first of these moments, for presenting possibilities of generalization until then unknown, and considered until today "one of the main sources of the usefulness of mathematics and the secret of its scientific power" (BELL, 2003, p. 18). According to Aristotle, "for Thales [...] the primordial question was not what we know, but how we know it" (BOYER, 1974, p. 33). According to Eves,

> At some point between Thales, 600 BC, and Euclid, 300 BC, the notion of logical discourse was established as a sequence of rigorous deductions from some initial assumptions explicitly enunciated […] no doubt one of the greatest contributions of the early Greeks was the development of this method of postulational reasoning (EVES, 2005, p.115).

According to D'Ambrosio, this preponderance occurs because mathematical knowledge, or Mathematics itself, was, since the Greeks,

> the most stable form of thought of the Mediterranean tradition that lasts until our days as a cultural manifestation that has been uncontested, to the other forms. […] Mathematics has become universal, displacing all modes of quantification, measurement, order, inference and serving as a basis, establishing itself as the logical and rational way of thinking that has come to identify the own species (D'AMBROSIO, 1990, p. 17).

There are countless examples of the use of Mathematics prior to the emergence of Greek civilization. However, the various manifestations in which mathematical elements appear, dating from before the 6th century B.C., are imbued with a practical, immediate sense. The emergence of demonstrations and deductions, due mainly to contemporary Thales and Pythagoras, is considered the milestone of the birth of mathematics, both by mathematicians,

> If all ancient civilizations had to develop, due to the needs of everyday life, processes for arithmetic calculation and measurement of quantities, only the Greeks, from the 6th century B.C., thought of analyzing the logical chains of such processes and thus created a completely new way of thinking (DIEUDONNÉ, 1990, p. 43).

and by mathematical historians:

> There is a gap between the practical empiricism of the surveyors who plotted the fields of ancient Egypt and the geometry of the Greeks of the 6th century BC. The former preceded mathematics; the latter, is mathematics itself. This gap establishes a bridge both with deductive reasoning applied consciously and deliberately, and with the practical inducements of daily life (BELL, 2003, p.14).

The historian Eric Bell, further on, states that "the essential part of its doctrine is the insistence on deductive demonstration" (BELL, 2003, p.19). Until then, there was, among the Egyptians and Babylonians, for example, a chain of problems solved inductively, in a practically particular way. Of course, we do not intend to deny the strength of inductive reasoning. However, it may present analogies that are only apparent and, therefore, lead to wrong conclusions. The method of deduction that comes with the Greeks provides reasoning, besides the generalization involved, with an incomparable capacity, based on the abstraction and detachment from reality impregnated in it.

If we take, for example, the definition 15 from Book I of the Elements of Euclid[1]

> A circle is a plane figure bounded by one curved line, and such that all straight lines drawn from a certain point within it to the bounding line are equal. The bounding line is called its circumference and the point, its center (EUCLID, 2009, p. 97).

or definition 14, from book XI: "When a semicircle with fixed diameter is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a sphere" (EUCLID, 2009, p. 482), we can observe that, instead of exemplifying by drawing a circle or a sphere, Euclid presents a general property that is sufficient to define such elements and only those elements.

The impact of the deductive method on the evolution of knowledge is not restricted to mathematical knowledge, but extends to the other sciences, because

> The axiomatic development of geometry has made a powerful impression on thinkers in the course of time, for the axiomatic form of geometry has affirmed itself to many generations of remarkable thinkers as the model of scientific knowledge at its best. It was natural, then, to ask whether other branches of thought, apart from geometry, can be situated on a

[1] The Book "The Elements", written by Euclid around 300 B.C., although more than two centuries after the time of Thales and Pythagoras, is considered until today one of the best models of the power of this deductive capacity, since "a relatively small number of axioms carry all the weight of the inexhaustibly numerous propositions derivable from them" (NAGEL AND NEWMAN, 1973, p. 14).

> reliable axiomatic foundation (NAGEL & NEWMAN, 1973, p. 15).

The second moment that we point out as historically important in this trajectory can be located from the middle of the Middle Ages, entering the Renaissance period, until the middle of the 17th century, and is marked by a process in which, in the end, one perceives again how, from the development of mathematics, the growth of the power of abstraction was used for advances in knowledge. On the other hand, it becomes visible how this development of mathematical knowledge also had as one of its causes the social context in which it was produced.

It would not be possible to delimit exactly the most important causes for the changes suffered by Europe in this period. It is a fact, however, that the needs relating to the accuracy of information and measures and the efficiency and speed of achieving results have increasingly emerged.

> During the late Middle Ages and the Renaissance, a new model of reality emerged in Europe. A quantitative model was beginning to replace the old qualitative model. Copernicus and Galileo, as well as the craftsmen who learned to make one good cannon after another, the cartographers who mapped the coasts of the newly discovered lands, the bureaucrats and businessmen who managed the new empires of the East and West Indian Companies, and the bankers who led and controlled the flows of the new wealth, all these people were thinking about reality in quantitative terms, more systematically than any other member of their kind[2] (CROSBY, 1997, p. 12-13).

Initially, the advent of cities, linked to growing commercial activity, brought with it a change in the nature of interactions between people, with each other, and between people and institutions.

> Cities have brought a new element to the European economy, until then confined in the narrow limits of an agrarian economy of small units [...] linked to the emergence of cities is the arrival, in Europe, of a new type of man, the merchant, very different from the type that existed there – his horizons are more torn, his interests are scattered in places very far from the continent, his psychological conditions harden and gain an audacity in the exercise of a profession in which the weak or those who love a quiet and sedentary life have no place (CARAÇA, 1958, p. 198).

The new reality, not only because of the emergence of cities, but also because of

---

[2] The author even credits the astonishing success of European imperialist expansion to the predominance of quantitative thinking and increasingly improved measurement practices, as opposed to the approaches that justify the success of this imperialism from ethnocentrism and social Darwinism.

the gradual and growing perception of the possibilities and potential of navigation and the military needs generated from it, has led to an increasing need for precision in measures. The measurement of elements such as time, space and heat could no longer be carried out in such an inappropriate or subjective manner. Dawn, sunset and weather were being replaced, as practical units to mark the weather, by the hours. First, the canonical hours, moments determined by the Catholic Church for prayers, in number of seven during the day, called Matins, Lauds, (the first prayers of the day), Third, Sixth and Noa (intermediate prayers), Vespers (late afternoon) and Compline (before bed, at night). Then, with the time measuring machines, i.e., the mechanical clocks, the practical unit of time measurement was gradually replaced by the clock hours. In 1370, for example, King Charles V of France decreed that all clocks in Paris would count the hours according to the clock he had installed in his palace. The final half of the Middle Ages, due to its marked characteristics of emerging changes, can therefore be marked as an extraordinary moment beginning a process of strong transformation in technique, knowledge, practices and society.

> From 1275 to 1325 someone built Europe's first mechanical clock and cannon, devices that obliged Europeans to think in terms of quantified time and space. Portolano marine charts, perspective painting, and double entry bookkeeping cannot be precisely dated because they were emerging techniques, not specific inventions, but we can say that the earliest surviving examples of all three date from that half century (CROSBY, 1997, p. 3031).

Although there are reports of mechanical clocks in China in the middle of the 10th century and of effective measurement systems used by peoples of Eastern countries, it is in Europe that an exaltation for measuring reality takes place.

> In the 14th century, when Oxford's Merton College scholars began to think about the benefits of measuring not only size, but also fleeting qualities like movement, light, warmth and color, they gave wings to the imagination and talked about quantifying certainty, virtue and grace. And indeed, if a person can think of measuring heat before the thermometer is invented, why should one presume to exclude certainty, virtue and grace? (CROSBY, 1997, p. 26).

It is not by chance that the reform of the Julian calendar occurred between the 15th and 16th centuries, despite the fact that the concerns about the discrepancies in relation to the date of Easter, pointed out as one of the main causes for the adoption of the Gregorian calendar, date back at least to the beginning of the 4th century. In the year 325, the differences between Christians of the West and the East regarding the date of the celebration of Easter were decided by the Council of Nicaea, led by Emperor Constantine, who set a universal date for the celebration of Easter, which would be the first Sunday after the full moon "that would take place at the spring equinox or immediately after" (LOPES, 2012, pp. 112-113). However, it is only at the end of the Middle Ages that social and economic demands impose the need for a more precise measurement and representation of time. In fact, the problem of reform was firmly attacked in this period, having been dealt with successively in the Councils of Constance (1414), Basel (1436 to 1439), St. John Lateran (1511 to 1515) and Trent (1545 to 1563), and the new calendar was effectively adopted only in 1582.

In a context of growing trade and a new urban reality, the search for new markets would bring new transformations. In the case of navigations, specifically in relation to Portugal, we see how the reciprocal influences between society and science appear again, since

> At the end of the 13th century, Portugal, which had already expelled the Moors and had become independent from Spain, was forced to seek commercial options across the Atlantic. Thus, the Portuguese vocation for navigation was defined, which effectively became a national project (D'AMBROSIO, 2008, p. 31).

Thus, at the beginning of the fifteenth century, Dom Henrique, called "The Navigator", using mathematics as an appropriate foundation for the nautical sciences, founded the School of Sagres, and propelled the "most important project of maritime expansion of humanity" (D'AMBROSIO, 2008, p.31). Despite being a project with very well-defined practical objectives – the expansion and consolidation of the Lusitanian domain – a certain degree of development of Portuguese science and mathematics was also achieved, since

> The ideal that inspired D. Henrique in the foundation of the Portuguese School of Astronomical Science is quite different from what motivated Alfonso X, King of Spain, in the foundation of the School of Toledo. The ideal of the Castilian king was purely philosophical; that of the Lusitanian infant was utilitarian. The scientific spirit was born among the Lusitanians later and rose high, but it was the navigations, with the problems they proposed to them and with the new aspects they presented to them of the phenomena of nature that created that spirit (TEIXEIRA, 1934, p. 19).

The new times and the new reality imposed (and also influenced) by the dominance of this quantitative direction also brought, together with the fact that "it is in the 14th century that the first firearms appear to be used in war" (VALENTE, 1999, p. 30), the need to improve the devices and war tactics, now with the help of more precise maps and proper mathematical tools such as tables and graphical representations, used to support the strategies to be adopted.

> Sixteenth century military manuals commonly included tables of squares and square roots to guide officers in the disposition of hundreds and even thousands of men in the new Renaissance Western combat formations: squares, triangles, tweezers, bastard squares, wide squares, and so on. The officers, when they were good, had to advance painfully through the wide sea of algebra and numbers, or recruit mathematicians to help them (CROSBY, 1997, p. 20).

The Art of War, a book written by Machiavelli in 1520, illustrates this condition, affirming that "just as a man dancing keeps time with the music and cannot make a false step, so an army properly observing the beat of its drums cannot be easily disordered" (MACHIAVELLI, 2006, p. 77). It is for no other reason that Rabelais, in 1534, describes the military maneuvers by characterizing them with an adjustment as precise as the mechanism of a clock,

> so well educated in military art, well-armed, well-disposed and following their insignia, ready to listen and obey their captains, expeditious in running, strong in fighting, prudent in adventure, that seemed more like an organ harmony or clock arrangement, than an army or a gendarmerie [3] (RABELAIS, 2003, p. 196).

The reference that both the Italian theorist and the French writer make, relating military exactness to the "rhythm of music" and the "organ harmony" should not go unnoticed, since, besides the fact that music was one of the elements of the Roman *quadrivium*[4], the standardization and slow stabilization of the representation of symbolic music, that is, of the staves and scores, with their specific codes, dates back to the 11th century, being, however, only consolidated in the Renaissance, being evidently associated with the more precise measurement of time. Thus, the choice that the Europeans undertook towards precision and efficiency,

---

[3] A word derived from the French gendarmerie, meaning Police Force.

[4] Geometry (resting forms), Astronomy (moving forms), Arithmetic (resting numbers) and Music (moving numbers), a group of four of the seven liberal arts taught in medieval universities (ATTIE, 2013, p. 44).

> extended even to what was less visual and more ephemeral, that is, music. On one page, you can see several minutes of music at once. You can't hear it, of course, but you can see it and acquire an instant knowledge of its entire bow through time. The Renaissance option in music was to limit variations, reduce improvisation. It made the same option in war, choreographing the acts of men in battle (CROSBY, 1997, p. 24).

Likewise, this "algebraic" effectiveness and precision of military maneuvers are also mentioned at the beginning of the following century, in 1603, in Shakespeare's writing, when Iago, after being despised by Othello in favor of Cassio for a military post, scorns his rival, calling him ironically "arithmetician" in clear opposition to the "algebraic" knowledge used by the competent officers:

> And what was he?
> Forsooth, a great arithmetician,
> That never set a squadron in the field,
> Nor the division of a battle knows
> (SHAKESPEARE, 1999, p. 14).

Thus, we see how this "total organic transformation" in society (CARAÇA, 1958, p. 202) brings with it the need for a tool that could quantitatively order this new reality. The effort to measure reality proved to be arduous, patient and slow, but the ongoing social and economic process still demanded some needs to be met, not only to measure, but also to explain and predict. The appropriate mathematical tool for this will be the function, conceived, refined and reworked over an extensive period of human history, and which will be "the actual spring that will touch all matters in these centuries[5] of creation of Europe and Modern Science" (CARAÇA, 1958, p. 203-204).

The emergence of the concept of function begins to be better delineated from the use of Indo-Arabic numerals. Due to their greater ease in calculations, we see a growing superiority of these numbers over the Romans[6]. Trade with Muslims, from the end of the 13th century, had provided the contact and knowledge of a numerical representation that was simpler and more powerful than the one known and used until then. In practical matters, let us consider the example of Francesco di Marco Datini, a merchant who lived in the 14th century and who, due to the increasing change in the nature and frequency of

[5] The author refers to a period that can be between the 14th and 17th centuries.
[6] Evidently, this dominance was not immediate. Still in 1447, for example, the painter Dirk Bouts placed on his altar a number represented with a confusing combination of the two systems: MCCCC4VII (CROSBY, 1997, p. 116).

commercial transactions, had been "driven to invent accounting in the same way that physicists were later moved to adopt calculation" (CROSBY, 1997, p. 189). The importance of this example lies in the fact that Datini recorded in a concise and meticulous manner a large part of his transactions, carried out in the 14th and 15th centuries, thus providing an overview of the evolution of his accounting records. Thus, we see that in 1366, a few Indo-Arabic numerals begin to appear in his ledgers, documents intended to record the financial and commercial movement. However, until 1383, these numbers are used in narrative form. Written this way, the data allows one to know a lot about the various characteristics of his transactions. However, the most important result, whether his company made a profit or not, does not appear immediately, and extensive calculations are necessary to arrive at this information. From 1383 on, however, the use of a system of columns, one next to the other, called "double entry", in which assets and liabilities were described separately, makes his accounting clear, by simply subtracting them to know the financial situation of the company.

In 1514 the artist and mathematician born in Bavaria (now Germany) Albrecht Dürer exhibits a painting called Melancholia, in which there is an example of a magic square, with Indo-Arabic numerals. The artist postulated that "the new art of the Renaissance should be based on science, in particular mathematics, because its precision and logic were very useful in matters involving proportion and graphic arrangement" (RONAN, 1987, p. 17).

The growing use of new numerals has brought with it the need for a change in operational notations. Leonardo Fibonacci, in the 13th century, already used the new numbers, but still struggled with the use of words instead of symbols. But by the second half of the 15th century the Italians were already using abbreviations for sum and subtraction operations. These abbreviations, however, could also cause errors, in case of algebraic equations. The currently known + (plus) and - (minus) signs, used respectively for sum and subtraction operations, appeared for the first time in print in Germany in 1489. Only in the following century, however, its use was adopted on a larger scale. The relationship between the adoption of the new numbers and the possibilities opened from that, for the development and consolidation of algebra and its symbolism is thus described by the mathematician Alfred North Whitehead, born in the 19th century,

> Arabic numerals have provided science with an almost perfect technical efficiency in handling numbers. The relief of the struggle with arithmetic details (as seen, for example, in the arithmetic of 1600 B.C.) gave way to a development that was already timidly anticipated in primitive Greek mathematics. Algebra has now come to the scene, and it is a generalization of arithmetic. Just as the notion of number is abstracted from reference to any set of entities, in algebra the notion of any determined number is abstracted (WHITEHEAD, 2006, p. 47).

Allied to the use of new numbers and the development of symbolic algebra, and even because of these, the beginning of the 17th century sees the appearance of Analytical Geometry, providing a powerful unification of two fields of mathematics, the geometric and the analytical, which have always been in separate compartments. The strength of the union between these two fields is provided by the fact that it makes it possible to combine geometric intuition with analytical reasoning, or, in other words, "... the importance of Descartes and Fermat's method comes from the fact that it makes it possible to translate any problem of plane geometry into an equivalent algebra problem" (DIEUDONNÉ, 1990, p. 67). Some mathematical elements have more than one type of representation, both algebraic and geometric, with a possible interchange of interpretation and resolution processes. As in music and war, a disturbing and potential-filled possibility arises of *seeing*, without having to do math, equations, tables of numbers and representations of quantities, sometimes associated with each other, on a page filled with curves and figures. And, on the contrary, it also became possible to interpret an algebraic representation and see there the geometric shape of those algebraic elements. "In this unification [...] lies one of the most dramatic, important and profound facts in the history of knowledge" (CARAÇA, 1958, p. 139), for this kind of two-way communication between two such different approaches until then makes Analytical Geometry become "a discipline born out of attempts to subject geometry problems to arithmetic analysis" (DANTZIG, 1970, p. 156-157), to become "the vehicle through which abstract properties of number were transmitted to the mind, giving the analysis a rich, picturesque language and directing it towards channels of generalization previously unimaginable" (DANTZIG, 1970, p. 157).

The conditions for the consolidation of the concept of function, whose importance is of capital order in this new type of company, are established.

> Navigation problems, for example, lead to an increasingly careful investigation of the movements of the stars and, in general, require a

> more rigorous study of the movement, a quantitative study, which allows measuring and predicting (CARAÇA, 1958, p. 199).

In this way, a step further is taken in the human process of understanding things. Observing and measuring the data of reality are very different capacities to perceive that certain processes obey laws and, more than that, that these laws could sometimes be understood and expressed mathematically. The long road to this point covers some necessary but not always sufficient steps. To perceive regularity in the phenomenon and the possible variables that influences its behavior. Then, use quantitative tools to try to relate the variables considered important. Next, qualify the relationship between these variables, that is, understand the mathematical law that covers the phenomenon. Finally, express this mathematical law of the phenomenon, or mathematize it. The crystallization of the concept of function thus emerges, on the one hand, from an illustrious descent of mathematical elements such as Indo-Arabic numerals, Symbolic Algebra and Analytic Geometry and, on the other hand, from the emerging social need for a quantitative ordering of reality. Thus,

> the new direction of science, which the new society determines and we see formulated in Da Vinci's writings, is the direction of a mathematical ordering of the universe. Later, in Newton's writings, this ideal of ordination will be formulated in polished terms: '... the modern world, rejecting substantial forms and hidden qualities, is concerned with referring natural phenomena to mathematical laws' (CARAÇA, 1958, p. 202).

From this context emerges what we consider the third fundamental moment in this process of valorization and institutionalization of mathematical knowledge in society, which would be marked by the predominance of mechanical philosophy. Thomas Hobbes, better known as a philosopher, but who was also a mathematician, tried in the 17th century to analyze politics and society with an approach through language and mathematics, which, according to him, would lead to a complete mechanistic understanding of the world. The certainty of mathematics would lead to correct and indisputable conclusions about society and man, "unlike the moral sciences which produced only endless controversies, an ethical vacuum" (KAYSER, 2006, p. 21).

In this context, the meeting of these three elements, the social conditions, Analytical Geometry and Symbolic Algebra, served as the main foundations for the emergence, from this century on, of more appropriate mathematical tools for the ordering of data (and its variations) and the explanation of phenomena.

> The initial structures of Differential and Integral Calculus date back to Archimedes' studies, and to the publication of these studies in Europe from 1550 onwards, followed by notes from various scholars, such as Stevin, Cavalieri, Barrow and Wallis, for example. However, it was only in the second half of the 17th century that Newton and Leibniz, using a series of previous results, achieved, separately, an accumulation of elements – such as functions (referred to as fluents by Newton), differentials, derivatives and integrals – in a single structure (ATTIE, 2013, p. 48-49).

The potential of Calculus tools in solving problems and discovering relationships between variables made the following centuries experience an incomparable scientific growth. The 18th century expanded the methods of Descartes, Newton and Leibniz into all the branches of mathematics that then existed. Not by chance, it was called the Age of Reason and also the Age of Enlightenment. It is the period in which the "Physical Sciences freed themselves from Theology [...] Science had revealed that absolute truth resided in Pure Mathematics, where, according to some, that is where it actually lies" (BELL, 2003, p. 376).

> The 18th century had been a brilliant period of intensive development of techniques introduced in the 17th century in mathematics, especially in analysis and its varied applications to other mathematical disciplines, such as geometry and the calculation of probabilities, mechanics and astronomy, with the success known for the prediction of natural phenomena (DIEUDONNÉ, 1990, p. 117).

The explanation of things from a purely mathematical or mechanical perspective – with functions, derivatives and differentials determining the behavior of events and predicting the quantities related to the variables of the phenomena – creates adequate cultural conditions, besides the existence of appropriate techniques, for inventions and discoveries. The publication by Lagrange, of his Analytical Mechanics, in 1788, provides a "direct and universal method, which unified the mechanics that existed until then, and which has been until today the most powerful instrument of the physical sciences" (BELL, 2003, p. 25). In this work, the French mathematician described the mechanical behavior of material systems based solely on scientific and mathematical principles, without the support of any spirit of nature[7]. The emergence of countless machines from then on was merely a matter of time, culminating in the process known as the Industrial Revolution.

> It is also due to 18th century mechanics – mainly due to Lagrange and Laplace – the speed with which machinery invaded civilization in the early 19th century [...] and mechanistic philosophy immediately shared its priceless benefits with the proletariat. Hundreds of thousands of people, who could attend the Lagrange and Laplace conferences for years without having learned a single thing, were meanwhile converted by the mute and infallible precision of their monotonous machines (BELL, 2003, p. 377).

Contradictorily, however, Dieudonné states that "under the Old Regime[8], only the schools devoted to educating the future officers provided a teaching of mathematics that gave a little room for infinitesimal calculation, and these were hardly accessible to the common people" (DIEUDONNÉ, 1990, p. 117).

In any case, it is right to consider that the emergence of steam machines, gigantic looms, locomotives, in short, the new apparatuses of the incipient Industrial Revolution[9] have brought about countless changes in the lives of individuals and institutions. Methods of administration in new factories, in new governments, an unprecedented urban population growth, changes in the physical structure and objectives of schools, etc. Another clear indication of this process of increasing rationalization of activities is the emergence, in 1745, of the first algebraic notation for the game of chess, proposed by Philip Stamma, "making it much easier to read the chess books, since the moves were described with words before that" (GIUSTI, 1999, p. 19). In all these changes, one of the few things that remains unchanged is the manifested presence of mathematical knowledge in each element of this new order, even though the understanding of how this presence takes place has diminished significantly since that period.

The deliberation about the length of the subway, besides having its origin in the ideals of the French Revolution at the end of the 18th century, also corresponds to this need

[7] Laplace was called by Napoleon for some clarifications, since it was customary to quote God on themes related to the mysteries of the universe. Napoleon would have asked Laplace: - Monsieur Laplace, you did not mention God even once in your book. Why? - Because I did not need this hypothesis (PENHA & SILVA, 2019, p. 212).

[8] Before the 1789 French Revolution.

[9] In 1801, for example, Joseph Marie Jacquard (1752-1824) completed the weaving machine with perforated cards, a device that would significantly influence ideas on how to operate a machine.

for accuracy and predictability in a society that is becoming increasingly rational, accelerated and complex. And this society now needs the development of mathematics to meet the growing demands of administrations, governments and civil, naval and military engineering. In the 17th century, we can refer to the cases of Hobbes, professor of Mathematics for the Prince of Wales and Newton, appointed inspector of the Royal Mint, and later promoted to director. But it is the 18th century that proves to be prodigal in governments keen enough to keep mathematicians under their wings, covering their expenses. Thus,

> Euler had been kept in the courts of Catherine and Frederick, Lagrange, by Frederick, Louis and Napoleon, Daniel Bernoulli (founder of Mathematical Physics), by Catherine. Monge and Laplace were employed by several successive French governments in posts of Military Engineering, training for engineers or administration, Fourier as well (BELL, 2003, p. 380).

In the following century, Napoleon Bonaparte would affirm that "the progress and improvement of mathematics are closely linked to the prosperity of the State" (BOYER, 1974, p. 344). From then on, the use of mathematical knowledge spread to all branches of the life of institutions and individuals. In the military field,

> the semi-empirical procedures of calculation, necessary for their practical usefulness in war, provided a complete mathematical prestige [...] it became clear that in modern conflicts it is difficult to destroy, cause damage or kill efficiently without a considerable use of mathematics (BOYER, 1974, p. 14).

In transportation,

> a nautical almanac is one of the indispensable things in modern navigation and, therefore, in commerce; machines are used, in general, for the hard work of performing the calculations; finally, these calculations depend on the movement of the planets and the latter are calculated from infinite series of numbers obtained by the Newtonian theory of gravitation (BOYER, 1974, p.17).

In governments and the economy,

> a human being can be as free as they were born, as a famous document state[10]; however, 130 million individuals are no longer as free as it was once imagined. Humanity as a mass is governed more despotically by the laws of probability than ever before by the decrees of any tyrant [...] to

[10] The author naturally refers to the Universal Declaration of Human Rights, proclaimed by the United Nations in 1948 and which states in its Article I that all people are born free and equal in dignity and rights.

> understand and analyze the reactions of the mass, whether those of atoms or human beings, one needs to master modern statistical methods. And the statistical method is the social mathematics per excellence (BOYER, 1974, p. 596).

Finally, the fourth moment highlighted by us comprises the period beginning in the 19th century, culminating with the advent of modern data processing machines. Aside from the enormous importance that certain contributions – mainly from Gauss and Cauchy – produced in the structuring of analysis and that the expansion of the power of abstraction, responsible for the emergence of new geometries[11] played in physical theories about the universe, perhaps the greatest influence that can be felt today in society from the development of mathematical knowledge from the ninth century is the emergence of computers. Resulting from a fruitful relationship between Logic and Algebra, this association arises[12] and is developed especially from the publication of the book "Mathematical Analysis of Logic", in 1847, by George Boole and also by the later works of Frege and De Morgan. What happens from then on is that the domain of abstraction would also extend to the principles of logic. The conversion of Logic into a type of Algebra, simple and easy to operate, is considered as one of the essential elements for the design of the electrical circuits that came to compose the computing machines. Already in 1870, Jevon's Logic Machine, which performed logical operations mechanically, was exhibited at the London Royal Society. The Turing Machines, proposed at the beginning of the 20th century, portray what was to come at the end of the century, an even more complete change in the lifestyle of individuals and societies.

In contemporary society, especially since the 1970s, social and economic valorization allied to newly created types of work that were always related to computer tasks was evident. Positions appeared in the job market with great demand and with remuneration above the average of that time, such as system analysts, programmers, operators and even drillers and typists.

There are currently not many fields of activity in which the human being is involved that have the prerogative of being immune to the use of data processors. Accounting, transportation, medicine, administration, school, government, arts, finance, war, sports,

[11] The geometries by Lobachevsky, Bolyai, Plücker, Riemann and Lie.

[12] Although it was already proposed by Leibniz in the 17th century.

personal relations, the press, toys, engineering, housing, in short, in most of the branches we can list, there is a considerable, if not indispensable, use of computers. In all these cases, the consequences, good or bad, of this true colonization by computers in the daily lives of individuals and institutions can be discussed. The fact is that the presence of data processors in the social body is evident and happens on a large scale, which certainly means an even more considerable existence and influence of mathematical knowledge sustaining this reality, even if one is not aware of the ways in which this sustaining occurs.

CONCLUSION

Regarding the four points of inflection mentioned here, we consider that they are sufficient to sustain the importance attributed to mathematics in our society. However, we also consider that the phenomenon of the co-existence of mathematics in all fields of human activity and the lack of understanding of mathematics by society in general is undeniable. Nevertheless,

> It is undeniable that there is considerable discourse and belief that mathematical knowledge is relevant in shaping the world. Even if one does not grasp which mathematics is used to produce today's reality, there is an intuitive and widespread perception that without mathematics almost nothing that exists would continue to exist as such. Just like the phenomenon of culture, anthropologically qualified as a component that, just like air, is not consciously perceived, even though it is around all people (ATTIE, 2013, p. 54).

Thus, a paradox is crystallizing: the greater the influence, the greater the invisibility! Even though the presence of mathematical knowledge in institutions, in daily life and in the production of goods is increasing, it also brings about an increasing invisibility of this kind of knowledge in these same spaces, that is, in institutions, in daily life and in the productive chain.

This invisibility appears as a result of the alienation processes of the individual – which, in the case of mathematics, are revealed in the face of the growing movement of abstraction of this knowledge and the removal of its processes of meaning – and influences not only the negative representations on mathematics, but also, in the case of school, the interactions between teacher and students.

Thus, in social terms, at the same time that the awareness of the importance of mathematics is increasing, an invisibility bubble is forming that is increasingly impenetrable in relation to its meanings. One of the most harmful consequences of the second phenomenon (the invisibility of mathematical knowledge) is the crystallization and perpetuation of this process of alienation of individuals from the universe around them and from their capacities and possibilities.

REFERENCES

ATTIE, J. P. **Relações de poder no processo de ensino e aprendizagem**. São Paulo, 2013. 164 p. Tese (Doutorado em Educação). Faculdade de Educação da Universidade de São Paulo, 2013.

BELL, E. T. **Historia de las Matemáticas**. México: Fondo de Cultura Económica, 2003.

BOYER, C. B. **História da Matemática.** São Paulo: Edgar Blucher, 1974.

CARAÇA, B. J. **Conceitos Fundamentais da Matemática**. Lisboa: Sá da Costa, 1958.

CROSBY, A. W. **A Mensuração da realidade***: a quantificação da sociedade ocidental.* São Paulo: Editora da UNESP, 1997.

D'AMBROSIO, U. **Etnomatemática:** *arte ou técnica de explicar e conhecer.* São Paulo: Ática, 1990.

D'AMBROSIO, U. **Uma história concisa da matemática no Brasil.** Petrópolis: Vozes, 2008.

DANTZIG, T. **Número: a linguagem da ciência**. Rio de Janeiro: Zahar, 1970.

DIEUDONNÉ, J. **A formação da matemática contemporânea.** Lisboa: Dom Quixote, 1990.

EUCLIDES. **Os Elementos.** São Paulo: Editora da UNESP, 2009.

EVES, H. **Introdução à História da Matemática**. Campinas: Editora da Unicamp, 2005.

GIUSTI, P. **História ilustrada do xadrez**. São Paulo: Annablume, 1999.

KAYSER, M. **A mecânica do desejo no encadeamento da ação no Leviatã de Thomas Hobbes.** São Leopoldo, RS. 120 p. Dissertação (Mestrado em Filosofia). Centro de Ciências Humanas, Unisinos, 2006.

LOPES, M. C. O. Calendário Atual: História, Algoritmos e Observações. **Millenium**, 43:107-125. 2012.

MAQUIAVEL, N. **A arte da guerra**. São Paulo: Martins Fontes, 2006.

NAGEL, E. & NEWMAN, J. R. **Prova de Gödel**. São Paulo: Perspectiva, 1973.

PENHA, R. M. & SILVA, M. J. P. Do Sensível ao Inteligível: novos rumos comunicacionais em saúde por meio do estudo da Teoria Quântica. **Revista da Escola de Enfermagem da USP,** 43(1): 208-214. 2009.

RABELAIS, F. **Gangântua e Pantagruel**. Belo Horizonte: Itatiaia, 2003.

RONAN, C. A. **História ilustrada da ciência.** vol. III. Rio de Janeiro: Jorge Zahar, 1987.

SHAKESPEARE, W. **Otelo, o mouro de Veneza.** Rio de Janeiro: Nova Aguilar, 1999.

TEIXEIRA, F. G. **Origem da cultura das matemáticas em Portugal**. Lisboa: Academia de Ciências de Lisboa, 1934.

VALENTE, W. R**. Uma história da matemática escolar no Brasil:** *(1730 – 1930)*. São Paulo. Tese (Doutorado em Educação). 211 p. Faculdade de Educação da Universidade de São Paulo, 1999.

WHITEHEAD, N. A. **A ciência e o mundo moderno**. São Paulo: Paulus, 2006.

Submetido em 29 de fevereiro de 2024.
Aprovado em 29 de agosto de 2024.