SOME DIFFICULTIES IN PROPOSING A MATHEMATICAL ISOMORPHISM TO INTERPRET WITTGENSTEIN'S *TRACTATUS*

Dedicated to Luiz Carlos Pereira in his 60th birthday

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ABSTRACT: Here I present some arguments against an alleged mathematical isomorphic relation between elementary propositions and states of affairs in the *Tractatus*. This is a standard proposal to understand the world-language relation in Wittgenstein's early philosophy. I advocate that the application of this type of isomorphism in the Tractarian conceptual framework conflicts with some of its philosophical peculiarities. This technical approach especially overlooks some crucial Tractarian tenets, such as a sharp modal distinction between form and structure and Wittgenstein's rejection of *a priori* true propositions.

KEYWORDS: *Tractatus*. Wittgenstein. Isomorphism. Language.

RESUMO: Aqui eu apresento alguns argumentos contra uma alegada relação de isomorfia matemática entre proposições elementares e estado de coisas no *Tractatus*. Esta é uma proposta tradicional para se entender a relação mundo-linguagem na filosofia do primeiro Wittgenstein. Defendo que a aplicação desta forma de isomorfismo à geografia conceitual do *Tractatus* conflita com algumas de suas peculiaridades filosóficas. Esta abordagem técnica negligencia especialmente algumas teses tractarianas cruciais, como uma distinção modal precisa entre forma e estrutura e a rejeição por Wittgenstein de proposições verdadeiras *a priori*.

PALAVRAS-CHAVE: *Tractatus*. Wittgenstein. Isomorfismo. Linguagem.

Introduction

There is no doubt that the Tractarian project has many problems. Indeed, the Tractatus as a philosophical program was abandoned by its own author who, over the course of two decades, developed an all-compassing self-criticism of his early presuppositions, images and arguments. Although such deconstruction can be held as unprecedented in the history of philosophy and very influential for the development of analytic philosophy itself, it falls outside the scope of this paper. Our approach here is internal to the *Tractatus* (TLP). For an adequate understanding of Wittgenstein's early philosophy, it is important to assume what Wittgenstein himself assumed at that time: that is, his concepts and presuppositions, without the introduction of his later (sometimes) distorted retrospective criticism (Kenny 1984).

In this contribution, we then address certain exegetical and technical problems concerning the application of mathematical isomorphism into the peculiar philosophy exposed in the *Tractatus*. In other words, what is really meant when it is argued that there is a mathematical isomorphism at the *Tractarian* foundation, since a projection of elementary propositions to states of affairs is indeed demanded there? To what extent is this type of technique an intrusion into a field guaranteed by a very special metaphysics? How could this approach help us understand the alleged elementary level of language? Or will this approach generate even more exegetical problems (conceptual or metaphysical) than we had before? Finally, what can we actually win (or lose) in the understanding of the *Tractatus* with this technical usage in its philosophical ground? In this sense, we aim to show how misleading it can be to demand a mathematical isomorphism in the exegesis of the *Tractatus*.

It is crucial to note that certain questions concerning a special connection between world and language in Wittgenstein's Tractarian period usually leads us to surreptitiously accept that we are addressing two separate, but actual, structures with different ontological status. These distinct structures, world and language, should (mysteriously) correspond to each other for our propositions to convey sense. This seems to justify the appeal to a seemingly harmless access route to the Tractarian

Bildkonzeption[1]: a discourse concerning isomorphism to account for a mirroring relationship between world and language. This exegetical key seems to be imported from certain mathematical procedures, and it is often justified by bringing more clarity to the behaviour of propositions as models and avoiding ambiguity, when sentences in the language are to be "connected" to their counterparts in the world. Certain points in this mathematical approach, however, bear little relation to a typically Tractarian concern: tracing the limits of the expression of legitimate language.

The very first meaningful evidence that makes that mathematical approach questionable is that nowhere in the Tractarian texts (*Notebooks 14-16* included) does Wittgenstein discuss a mathematical isomorphism. Furthermore, it is doubtful that we need such a mathematical technique for better understanding the relation between world and language in Wittgenstein's first masterpiece. The present contribution is divided into two sections. The first section discusses how plausible the application of a mathematical isomorphism in Wittgenstein's first book may actually be. The second section presents certain arguments against the technical use of isomorphism in the *Tractatus*.

1. The Plausibility of Isomorphism in the *Tractatus*

In the Tractarian passage 5, we have a clear assumption of a rigid truth-functionality in the composition of language, *i.e.*, complex sense and truth should be *fully* established by the sense and truth of elementary propositions that comprise them. For this, we need elementary propositions that follow the logical multiplicity of atomic facts. Furthermore, simple objects are articulated in states of affairs or facts. These special objects should then be eternal because they can neither be generated nor disintegrated. This peculiar ontology comes to fulfil the conditions of representation presented in the famous Tractarian *Bildkonzeption*. There we have general representational conditions that are organised and perfectly traceable (TLP 2.1-2.225). The representation must be a complex, a fact, and not a simple object or element. One complex must be correlated to another (TLP 2.1511-2.1515). A complex that represents

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1 Because it is highly problematic to maintain that the *Tractatus* has any type of theory, in this paper we use the term "pictorial conception" or "*Bildkonzeption*" and not "picture theory" or "*Bildtheorie*". Moreover, here the relevant Tractarian difference between complex and simple are to be discussed at the elementary level of language. Of course, atomic propositions and atomic facts are atomic in a way that is different from the way in which Tractarian names and objects are atomic. For instance, the complexity of aRb is clearly different from the complexity of p.q. Here we deal exclusively with the first kind of complexity.

must possess the same logical multiplicity of the represented complex, *i.e.,* it must be possible to identify in the complex that represents as many elements as there are to be distinguished in the represented complex. Here I use 'complex', a more general and neutral word, because I do not intend to make a statement concerning the distinction between *Tatsache* and *Sachverhalt*[2]. Rather, the vagueness of the term 'complex' is unproblematic in our account because this term should cover both ontological complexes and linguistic complexes.

There should be a direction in this representation to prevent symmetry in a representational relation. Because here we have the basis for my main criticism to come against the usage of mathematical isomorphism in the *Tractatus*, we must more carefully examine this asymmetry or directedness in the pictorial relation. Assume that a representational relationship is a binary relation in which a complex X represents or depicts a complex Y. In other words, if Y is indeed the complex represented by another complex X, it must be discarded, according to the Tractarian conditions of representation, that Y also represents X. If Y is also a representation of X, Y should also be a fact because only an actual complex, that is, a fact, (and not just a mere possible arrangement of things) can represent something. This is crucial: all representations in the Tractarian *Bildkonzeption* are facts in the world (2.141). Symmetry in the pictorial relation would then make X, as well as Y, always true because Y should also be a fact; Y should be an actual structure in the world in order to represent X.

However, a picture should never be *a priori* true (2.223-2.225). A description (or a contingent proposition) cannot be *a priori* true because its truth must be determined by a comparison with a portion of reality (6.113). Wittgenstein consequently makes use of a certain concept of surrogative reasoning in the context of his *Bildkonzeption* to avoid *inter alia* this type of problem. As a result, the elements of the representation must stand for (*vertreten*, substitute) the components of the represented complex (2.131). The elements of the representation replace the components of the represented complex in the representation *and not the other way around.* This last condition blocks a reversal in the pictorial relation, that is, if X represents Y, Y cannot represent X, if we have a *direction*

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[2]In a valuable discussion, Oscar Marques (1991) critically responds to Edgar Marques' (1990) modal account of the distinction between *Sachverhalte* and *Tatsache* in the *Tractatus*. We do not address this contention here, but I hope my criticism of the application of mathematical isomorphism in the *Tractatus* shall show how some notion of possibility is necessary to understand the so-called Tractarian ontology and *Bildkonzeption*. Neither O. Marques nor E. Marques address the distinction between the form of a representation and its structure in their works. Even if some possible *Sachverhalt* is not required to determine the truth conditions of a proposition, as O. Marques argues, we need some notion of possibility to understand the asymmetry between a proposition and the state of affairs in the pictorial relation.

in the representation marked by the notion of substitution (*Vertretung*). In this sense, the assumption that this relation should be symmetrical is incompatible with the *Tractatus*[3].

In addition, a complex of elements without organisation cannot represent nor be represented. No list or mere collection of things would sufficiently constitute a representation; only an articulated complex, or, in one word, a fact, should be taken as a legitimate representation. Whenever we discuss a complex, we are discussing organised complexes[4], not plain sets. This point is crucial for certain of our arguments here. The young Wittgenstein calls an *actual* articulation of elements a "structure of the representation" (*Struktur der Abbildung*), and he calls the *possibility* of this articulation a "form of representation" (*Form der Abbildung*). In conclusion, we have the last and most important step in this Tractarian *Bildkonzeption* (TLP 2.15): the representation and the represented complex must bear the same articulatory possibilities for a representational relation to be established. In Tractarian jargon, representation and the represented complex must have the same logical form (*dieselbe logische Form*[5]).

It is important to recall in a more direct way the Tractarian conditions necessary for something to be taken as a representation. If something is a representation, 1) it must be a complex; 2) it must be a complex correlated to another complex; 3) they must have the same logical multiplicity; 4) there must be a direction determined by the substitution, in the representation, of the represented complex's elements, that is, the pictorial relation should not be symmetrical; 5) it must have a determined structure (*i.e.,* it is not a collection of things); 6) it must have a form; and 7) its logical form must be identical to that of the represented complex.[6]

It is easy to see what Wittgenstein was aiming at with the last clause. When he

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[3] Anscombe (1995) briefly discusses this problem (but without my argument): "The problem was constituted by the isomorphism's being two-way. If a figure x is isomorphic with a figure y, then equally y is isomorphic with x. So how does x's isomorphism with y show that x is a picture of y any more than y is a picture of x?" (p. 399). I do not agree that the argument of isomorphism's being two way fails. In what follows, I will show some reason.

[4] Note that to discuss an actual structure of elements does not commit us to some Platonic structures. The type of realism in *Tractatus*, if we actually have some version of realism there, must be other than Platonist.

[5] This necessary point is instructively to be found in Hertz' *Bildkonzeption*, as well: „Wir machen uns innere Scheinbilder oder Symbole der äußere Gegenstände, und zwar machen wir sie von solcher Art, dass die denknotwendigen Folgen der Bilder stets wieder die Bilder seien von den Naturnotwendigen Folgen der abgebildeten Gegenstände. Damit diese Forderung überhaupt erfüllbar sei, müssen gewisse Übereinstimmungen vorhanden sein zwischen der Natur und unserem Geiste. Die Erfahrung lehrt uns, dass die Forderung erfüllbar ist und dass also solche Übereinstimmungen in der Tat bestehen." (1894, p. 1)

[6] At this point, we have a seminal instance of the typical modal movement found repeatedly in the *Tractatus*. It is clear here, as by some system of modal logic, that if p is possible, p must necessarily be possible. In the relation between the above items 5 and 6, if a complex has a structure, then this structure is necessarily possible. The possibility of articulation is necessary. As a result, if a complex has a structure, this structure *must* be necessarily possible. This is also to be found in the Tractarian bipolarity as a criterion of meaningfulness: a proposition must exhibit the two poles, true and false. That is, a proposition *must be able* to be true *and must be able* to be false too.

insisted that the elements of the representation must have the *same* possible relationships as the elements of the represented complex, he wanted to gain a sense of surrogative function[7] or, rather, of a preservation of relations from one complex to another by the conservative substitution of elements and relations of the represented complex by elements of the representation. For this extrapolation to be legitimate, it should be possible to recognise the same combinatorial possibilities in the two complexes, the same logical form, and, then, the same possible relationships between their components. In this way, by considering a legitimate proposition as a complex that represents, and the fact that makes it true as the represented complex, the peculiar Tractarian metaphysics fully justifies this preservative projection from the proposition to the complex that makes it true.

Nevertheless, what we have with the recurrent proposal of an isomorphism in the *Tractatus* does not seem to be an unproblematic suggestion of a common form, *iso* (same) and *morphè* (form), but something stronger. We have the suggestion of a conservative one-to-one mathematical correspondence of the form of a structured complex in another structured complex. For example, Hintikka and Hintikka (1986) state: "The crucial idea in Wittgenstein's view is precisely what mathematicians mean by an isomorphic representation or isomorphic mapping" (p. 93).

We can note another example of this mathematical proposal in Stegmüller's (1969) presentation of the *Tractatus*. Stegmüller claims:

> "There are thus two requirements for this sort of isomorphism between two complex facts F and F'. (1) there must be a one-one mapping between the individuals and between the equal-place attributes; (2) based on a mapping of this sort between categorically like elements of F and F', there must be a one-one correspondence between the simple states of affairs of F and F' such that a state of affairs in F exists if and only if the state of affairs form F' that is paired with it under this correspondence exists." (Stegmüller, p. 407)

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7   In his paper on structural representation, Swoyer (1991) revealingly maintains: "Structural representation has a distinguished philosophical history. Leibniz called it *expression,* telling us that one thing "expresses something in which there are relations that correspond to the relations of the thing expressed", so that "we can pass from a consideration of the relations in the-expression to knowledge of the corresponding properties of the thing expressed". Apart from his claim that each monad expresses the entire universe, many of Leibniz's examples are quite similar to the examples of structural representation discussed below: a map of a region expresses the region, a model of a machine expresses the machine, the perspectival projection of figure on a plane expresses *the* original figure, speech expresses thought. The view that something very like structural representation underlies such diverse phenomena as the semantics of natural languages and the propositional attitudes form the core of Wittgenstein's picture theory in the *Tractatus* (1921). (...)Wittgenstein was much closer to the truth than is commonly supposed, and it is natural to conjecture that the glaring defects in his account derive mainly from its accompanying doctrines of extensionalism, logical atomism, the absolute simplicity of objects and (arguably) nominalism: none of which have any part in the present story" (p.52).

As we can see, these exegetical proposals claim not only that we have a common form between elementary propositions and states of affairs but also that we have a certain relation that should be nicely captured by a technical isomorphism between elementary propositions and states of affairs. This would turn out to be, as these authors hold, a necessary condition for understanding the Tractarian conceptual framework itself.

We can identify Stenius as the father of this application of isomorphism in Tractarian fields. He introduced it in the 1960s to avoid certain exegetical and conceptual difficulties in relation to Wittgenstein's first work. This is precisely the decade in which the notion of isomorphism gained traction as a relevant technique in mathematics and logic. Stenius (1960) offers what he calls "a system of reference" to understand Wittgenstein's first philosophy, which has indeed become very influential:

> "Wittgenstein's use of the notion of representation (Abbildung) and related concepts is not free from a certain amount of ambiguity, and I do not think it is possible to grasp exactly what idea he connects with them. I shall therefore adopt the following method of analysis. First I shall define an exact concept of 'representation' called isomorphic representation. Then I shall interpret Wittgenstein's statements about pictures with reference to this sort of depicting. In this way we obtain a model that satisfies many of Wittgenstein's statements on the subject. This model can be used as a system of reference for the analysis of Wittgenstein´s application of the concept of a picture in his theory of language." (p. 91)[8]

It is an attractive fact that the technique of mathematical isomorphism seems to capture in certain lines what Wittgenstein intended in his proposal of a pictorial representation between propositions and states of affairs, namely: 1) naming at the elementary level of language cannot be ambiguous, *i.e.*, it cannot be the case that we have one name referring to two objects of the complex that makes the correspondent elementary proposition true. In fact, this demand could be satisfied by the functional aspect of a mathematical isomorphic mapping, *i.e.,* each element of the image set must be connected to one, and only one, element of the complex; 2) Tractarian naming does not allow synonyms, *i.e.*, at the elementary level of Tractarian "semantics" there could not be anything like an object that is designated by two names, a requirement that is evident as a consequence of the rejection of identity in the Tractarian notation (TLP 5.53

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[8] We can also note some echo of this application of technical isomorphism in the influential *Companion* by Black (1964, pp. 68-9). However, the idea of an isomorphism of names, as presented, for example, by Black (1964, pp. 68-9), is demonstrably false because we need complexity for the preservative mathematical mapping and because names are simple in the *Tractatus* (cf. 3.2-3.21). It is important to emphasise here that there is no structure-free name in the *Tractatus*. In this way, it is clear that if we should defend an alleged isomorphism in Wittgenstein's first book, this should not be observed between names and simple objects, but instead between structures composed by these objects. We need structures for representation, not a list of isolated elements. Naming has to be made in a structure, within propositions. Therefore, naming should always be, in a sense, holistic, *i.e.*, it always depends on the structure wherein the name is necessarily inserted. There are no names or objects therein that are isolated or without a structure in which they are inserted.

and 3.325). In the Tractarian metaphysics of symbolism, to say that "x = y" is a nonsense and to say that "it is not the case that x=y" is a triviality. This demand could be satisfied by the injective aspect of a mathematical isomorphic mapping: each object of the whole domain can have only one element of the image set assigned to it; 3) Tractarian naming does not allow for an object in a complex represented by an elementary proposition to exist without being designated by a name, *i.e.*, all objects in an structured complex should have a name at the elementary level of language. In other words, the elementary proposition should fully cover the ontological complex that should make it true. In set-theoretical jargon, the names in an elementary proposition must map all the objects of the depicted complex for the proposition to have its sense determined. This demand could then be satisfied by a surjective isomorphic mapping: every element of the domain finds its counterpart in the image set, *i.e.*, there is no element in the domain that does not have its image given by the function.

Furthermore, we may also have some instance of surrogative reasoning (or by-substitution reasoning), which is proper to the building of models. From a model, one should be able to read off everything (or the relevant things) that occur within the complex represented by this model. This should happen because the elements in the model replace the elements of the represented complex. The relationship between the structural representation and what is represented must be such that we can "read" the latter by the former. To this end, by formulating it in Tractarian jargon, the two structured complexes must have the same logical form, that is, the same horizon of possible articulation. Therefore, through the pictorial relation it should be possible to systematically preserve the formal characteristics of the elements depicted in the elements of the representation. Nonetheless, how is this done in the *Tractatus*? Its peculiar metaphysics works well to meet the demand of conservation between structured domains.

In passage 5.4711, which succinctly shows much of the project of the *Tractatus*, it can be argued that there is a type of transitivity between domains[9]. It is easy to force a mathematical isomorphism into this picture, including the possibility of surrogative reasoning: if one understands a proposition as a structure and the state of affairs that makes it true as another structure, the preservation of relations of one in another could occur by the very definition of mathematical isomorphism. This would work without the

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9   Here, transitivity must be understood as an instance of *Übertragung*, as an extension, extrapolation or transmission, rather than in terms of the technical meaning of transitivity.

need to venture into the onerous Tractarian metaphysics, *i.e.*, without mention of the essence of language and world and without mention of an absolute logical space. Put roughly, that which might be solved by the contention of the peculiar metaphysics of the *Tractatus*, which leads to the negative diagnosis of the whole philosophy, could then be replaced by the contention of a mathematical technique that gives us everything that we could want, namely: non-ambiguity, non-synonymy, full-covering and preservation between structures.

Here a question should be asked: what could we actually gain when we address the Tractarian *Bildkonzeption* as incorporating a mathematical isomorphism, *i.e.,* a one-to-one preservative correspondence between two structured domains? The positive results of this interpretation seem to be attractive enough to encourage and justify this application, which has become almost commonplace among interpreters of the *Tractatus*. From it, we could gain some technical instrumentality with clearer criteria for identifying and applying this two-way preservative correspondence. Additionally, with this we could have greater clarity regarding what can be understood by projection in the *Tractatus*. Furthermore, we could gain a more detailed understanding of the functioning of models themselves in the Tractarian *Bildkonzeption*.

Nonetheless, there is, conversely, also room for us to ask what we can lose by applying a mathematical technique to a philosophical system, especially to the peculiar metaphysical claims that are made in the *Tractatus*. Common examples of Tractarian metaphysical requirements are the existence of eternal and simple objects to determine an absolute logical space of possibilities or even the existence of essentially simple names comprising logically independent elementary propositions. The answer to the question of potential losses shows us that we lose more than we should accept with this mathematical-isomorphism-interpretation in the *Tractatus*. This leads to inevitable conceptual difficulties by distorting certain central Tractarian tenets.

2. Certain arguments against the application of mathematical isomorphism to the *Tractatus*

A first point to be observed is that seeing isomorphism in the *Tractatus* is not appropriate from a plain meta-theoretical standpoint: it is anachronistic. At no time did Wittgenstein ever make use of this term in any of his works, including in his mature phase, when the technique of isomorphic mapping seems to be already well established

in mathematical practices. This absence of 'isomorphism' in his writings is easy to be checked after the availability of Wittgenstein's *Nachlass*. In any case, it would be enough to find that this term has never been used in any of his preparative or more finalised texts or in his canonical works. Indeed, it is plausible to doubt that Wittgenstein possessed any direct technical knowledge of isomorphism and its utility in mathematics to explain the systematic transformation between abstract structures.

A second and more serious problem is that, even if the isomorphism between structures properly expresses what Wittgenstein wanted with the remission of a meaningful proposition to another portion of reality that makes it true, one very often neglects how this relatively simple mathematical technique would affect the Tractarian conceptual framework. Roughly, it would seriously conflict with certain peculiar metaphysics underlying the *Tractatus*, *i.e.*, it would make certain underlying metaphysics, typical of the *Tractatus*, irrelevant for the allegedly full determination of propositional sense.

There would be, for example, no reason for the requirement of a peculiar ontology of ultimate, simple elements in reality for propositions that have a determined sense (TLP 2.02-2.0212). This is so because, by thinking in terms of a mathematical mapping between structures, elements of a mapped domain can be chosen arbitrarily according to some contextual relevance and pragmatic purposes. This pragmatism makes it possible for the structures to be mapped via some mathematical isomorphism in which their relations and properties would be preserved without any mention of any grounding metaphysical account. In spite of the irrelevance of any metaphysical motivation in implementing a mathematical isomorphism, the *ad hoc* selection of elements in a structure to be explored, or even the *ad hoc* selection of the structures themselves, clearly presupposes the possibility of meta-language, which is "vetoed" in the *Tractatus*: we cannot leave language to talk about language. Nothing like a hierarchy of languages can be expected in the *Tractatus*.

States of affairs are all comprised of the peculiar Tractarian objects, which are *essentially* simple. In this sense, we must be cautious with what we call 'simple' in applying a mathematical isomorphism to the *Tractatus*. Simplicity in the *Tractatus* must be logically understood for determining the (radical) limits of discourse analysis. When we say 'logically simple names', we should mean names that do not (and could not) involve any type of abbreviation or definition. When we say 'logically simple objects', we should mean objects that no longer accept descriptions. We do not need this type of

essential simplicity to undertake any mathematical isomorphism between two structures. In this context, we would only require that elements of both structures should be atomic, in the sense that, in this context, they cannot be "broken down" or analysed further. That is, they would be, by some definition, the end of the analysis. The restriction would be contextual and pragmatic rather than metaphysical or essential. We would no longer need eternal and indestructible objects because any element (be it empirical, macro- or microscopic, or abstract, such as a number or a concept) could be taken as an object in a structure to be represented or in a structure to represent another one.

Another problematic consequence, correlative to the one above, would be that we would have some arbitrariness of internal relations because the projection of elements of a complex onto elements of another complex, required by the Tractarian *Bildkonzeption*, could be determined by choices with certain pragmatic aims. In this application of mathematical isomorphism, we would also have "surrogativity" according to some definitions, as Swoyer (1994) shows. We could systematically define which relationships or properties of a complex should be preserved in another complex, thus relying again on the meta-language explicitly condemned in the *Tractatus*.

Still, the conservation of the horizon of combinations between the elements of a proposition and the elements of a state of affairs, essential to the *Tractatus*, as noted in the former section, would not have to be given by any necessary mirroring between the essence of language and the essence of the world. This logical mirroring could be promptly and artificially established by the isomorphic preservation of one mathematical structure in another, if we are allowed to hold a proposition as a structure of names and a state of affairs as a structure of objects. By definition, the application of the isomorphism to the *Tractatus* implies what the early Wittgenstein seemed to want: a structure (such as language) is sent to another (such as the world) in a one-to-one manner, *i.e.*, injectively and surjectively, so that relations between the elements of one (e.g., objects) are preserved by the elements of the other (e.g., names).

Wittgenstein himself stated that he partially inherited the concept of *Bild* from mathematics. The ground for this use is to be already found in the origins of this notion in Wittgenstein's thought, as conversations with the Vienna Circle draw attention to:

"I have inherited this concept of a picture from two sides: first from a drawn picture, second from the picture of a mathematician, which already is a general concept. For a mathematician talks of picturing in cases where a painter would no longer use this expression. The word 'picture' has one

> advantage: it has helped me and many other people to make something clear by indicating a common feature and pointing out: 'So that is what matters!' We then have the feeling, 'Aha! Now I see, a proposition and a picture are of the same kind.'(…)" (Waismann 1979, p. 185)[10]

A contemporary example of the use (or misuse) of this technique is found institutionalised in *The Blackwell Dictionary of Western Philosophy* (Bunin and Yu, 2004). This shows how far the use of isomorphism in the *Tractatus* has become firmly rooted - not only in the secondary literature about the *Tractatus*, but in the secondary literature on philosophy itself. Indeed, Bunin and Yu (2004) define isomorphism in the following way, using the *Tractatus* as an unequivocal example. For them, isomorphism is "the structural identity or one-to-one correspondence of properties between two propositions or two systems. In Wittgenstein's *Tractatus*, a proposition is a picture that is isomorphic with a corresponding possible state of affairs. This is the core point in his picture theory" (p. 362).

It must be said that this definition of isomorphism includes a technical problem in that it states that a correspondence between two systems can be called an isomorphism. This condition is necessary but not sufficient; we also need the very important notion of the *preservation* of relations or predicates between the components of these systems. The one-to-one correspondence between two structures is not enough to make them both isomorphic. Moreover, Bunin and Yu also use certain notions without finer distinctions, which are necessary to understand certain decisive tenets in the *Tractatus*. For instance, it is far from obvious to understand what should be an isomorphism between a proposition, which could be taken as a structure, and a *possible* state of affair, which cannot be a structure in the same sense in which a proposition is a structure. After all, should isomorphism be defined in terms of forms or in terms of structures? There is, as we saw above, a relevant difference between forms and structures in the *Tractatus* (2.15). Is it then possible to have isomorphism between the form of one system and the structure of another or vice versa, as Bunin and Yu seem to argue?

The difficulty in answering this question shows a serious terminological confusion between structures and forms in interpreting isomorphic mapping of facts in the *Tractatus*. This persistent difficulty, in turn, has resulted in a conceptual confusion between modalities, *i.e.*, the sharp distinction between the *possibility* and the *actuality* of structures has become blurred.

An *actual* complex cannot represent another *actual* complex as a condition for its

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10 It is very difficult to understand why Wittgenstein does not mention Hertz here.

meaningfulness. Because we are addressing the determination of sense and not the determination of truth, we cannot have *a priori* true propositions. The mirroring between *structures* is a direct consequence of demanding the relationship between representations in using a mathematical isomorphism. With the analogy to a mirror we clearly have two *actual complexes* or, in a Tractarian sense, two structures.

Rather, symmetry makes sense in the mathematical definition: if we have an isomorphic relation between two structures, we do not have to know what represents what, or which complex indeed represents the other. The point is that, in a mathematical isomorphism, we must have two complexes (structures) in order to have this *two-way* preserving projection. However, in the *Tractatus*, the second complex, the one being depicted, should have another *modal* status. This complex to be represented must indeed be a complex, but not a *structure*, because it must be just *possible* if we are concerned with the determination of sense, or in Tractarian jargon, it must be a form (2.15). The second step after the determination of sense would be the determination of truth value, or in other words, checking if the former represented (possible) complex is indeed an actual complex or structure (fact) in the world.

What is at stake here is the basic argument with which Wittgenstein essentially seems to try to solve all philosophical problems: sense and necessity must be different, and moreover, mutually exclusive, incompatible notions. To say something meaningful is to say something contingent. To say something necessary means abandoning the ground of sense. In this sense, to state that there is an isomorphic relation between two facts or between two actualised structures is unattractive to the *Tractatus*. With his *Bildkonzeption*, Wittgenstein tries to account for conditions for which a proposition has sense, an initial and mandatory step for it to be true (or false). The concern and commitment of the *Tractatus*, prior to the determination of truth, is to the determination of the sense of propositions. The logical ground for the constitution of a sense of our discourse is that it should (logically) precede the empirical ground of the determination of its truth value.

If representations could correspond symmetrically to their represented complex, in a two-way preserving remission, we would be in possession of their truth, just by determining their sense, because the represented complex would be actual in the world, or using Tractarian jargon, would be, as a result, a structure or a fact. We must recall that only facts could depict something.

As we have already discussed, there should be a *direction* in picturing to prevent

symmetry in a representational relationship. This direction marks the asymmetry that we are proposing to understand the Tractarian pictorial relation. Wittgenstein's use of the notion of substitution in his *Bildkonzeption* is highly consequential: the elements of the representation should stand for (*vertreten*) the components of the represented complex. The elements of the representation *replace* the components of the represented complex in the representation *and not the other way around.*

We have here a terminological trap in the context of a modal question. In the *Tractatus*, we have form (*i.e.,* the combinatorial possibility between elements) being preserved (TLP 2.15); while in mathematics, we have structures being preserved (*i.e.,* the very articulation or arrangement of the elements is actual). Problems arise through isomorphic relation being symmetrical: for example, if a representation A is mathematically isomorphic with a represented complex B, then B is also mathematically isomorphic with A. In this sense, the reversibility of bijection must be held as a central problem of applying a mathematical isomorphism to the *Tractatus* because a mathematical isomorphism commits us to at least two *actual* structures in a *symmetrical* relation. This type of bijection is a necessary condition for mathematical isomorphism, but it cannot exist in the *Tractatus* because, for the young Wittgenstein, pictorial relation has a direction, that is, it should be *asymmetrical*: if A represents B, B could not represent A.

Another difficulty must be faced. It is important to note that the pictorial relation, which would be represented by the arrows between structures in the presentation of a mathematical isomorphism, cannot be a third thing that connects two independent structures, if we apply it to the *Tractatus*. In set theory, we could try to represent a set A mapping B as, say, A → B, but in passage 3.13, we note that this representation of mapping in the *Tractatus* should be more reliably represented as, say, (A →) B because the represented complex does not belong to the complex that represents. This represented complex should be outside[11] of the complex that represents. The pictorial relations belong to the complex that represents (here, A). Thus, the reversal function, implied by the reading of isomorphism, should show that we would be at last in a different domain of representation, namely: A (← B) or (B →) A, where pictorial relations belong to B and not to A. This proposed schema highlights that Tractarian

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11 It is important to emphasize here that this "outside" is relative to the complex that represents, not in respect to world: a proposition is a fact within the world that may represent other facts. Obviously, a proposition should be outside these facts that it represents, but not absolutely outside of the world.

relations of representation are *asymmetric* by showing the scope of the arrow and to which complex the pictorial relations indeed belong.

Conclusion

The discourse concerning a mathematical isomorphism in the *Tractatus* motivates us to consider the extent to which our questions may adversely impact our answers. In general, our questions determine the possible horizon of legitimate answers. Conversely, inappropriate questions can lead us away (either completely or to a significant degree) from understanding what we had wanted to understand before formulating them. An illegitimate question may also contaminate its answer, leading to conceptual distortion.

In this sense, the recurrent and sometimes consensual suggestion of mathematical isomorphism must continue to be approached as only a suggestion or instructive image. It is as useful (and risky!) as thinking about the Tractarian *Bildkonzeption* with everyday empirical objects. Associating a certain technical isomorphism with the *Tractatus* means preparing certain pigeonholes that represent certain of our own misconceptions.

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